

# Community Standards

Alan D. Miller\*

August 14, 2013

## Abstract

I introduce a model of community standards relevant to the judicial determination of obscenity. Standards are defined as subjective judgments restricted only by a simple reasonableness condition. Individual standards are aggregated to form the community standard. Several axioms reflect legal concerns. These require that the community standard (a) preserve unanimous agreements, (b) become more permissive when all individuals become more permissive, and not discriminate, *ex ante*, (c) between individuals and (d) between works. I show that any rule which satisfies these properties must be “similar” to *unanimity rule*. I also consider explore the relationship between the model and the doctrinal paradox of Kornhauser and Sager (1986).

*Keywords:* Community Standards, Axioms, Obscenity, Aggregation, Doctrinal Paradox

*JEL classification:* D70, D71, D72

## 1 Introduction

In the United States, material deemed to be obscene according to “contemporary community standards” is not protected by the freedom of speech and is generally criminalized.<sup>1</sup> I introduce a new model in which community standards are formed by aggregating a set of individual standards. In the model, standards are defined as judgments—categorizations of possible works as either “obscene” or “not obscene.”

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\*Faculty of Law and Department of Economics, University of Haifa, Mount Carmel, Haifa, 31905, Israel. Email: admiller@econ.haifa.ac.il. Web: <http://econ.haifa.ac.il/~admiller/>. Phone: +972.52.790.3793. Fax: +972.4.824.0681. Helpful comments were provided by an associate editor as well as Christopher P. Chambers, Federico Echenique, Jon X Eguia, Shuky Ehrenberg, John Geanakoplos, Henry Hansman, Sergiu Hart, Philip T. Hoffman, Matias Iaryczower, Leo Katz, D. Roderick Kiewiet, Michael Mandler, Eric Maskin, R. Preston McAfee, Stuart McDonald, Li Nan, Robert Ostling, Ben Polak, Dov Samet, Eran Shmaya, Matthew Spitzer, Yair Tauman, Oscar Volij, David Wettstein, William Zame and seminar participants at the California Institute of Technology and at the Eighteenth International Conference on Game Theory. All errors are my own.

<sup>1</sup>See *Miller v. California* 413 U.S. 15 (1973) and 18 U.S.C. §§1460–1470.

Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider all works to be obscene. I define several basic normative properties of aggregation methods which reflect legal concerns expressed by the judiciary. I then show that any method satisfying these properties must be “similar” to *unanimity rule*, in which a work is considered obscene if and only if all community members consider it to be obscene.<sup>2</sup>

The basic model can be described as follows. First, there is a community, which can be any group of individuals. The Supreme Court has required that the community be defined in geographic terms and contain all adults in that community, including the young, the old, the religious, the irreligious, the sensitive, and the insensitive.

Next, there is a finite set of all possible works.<sup>3</sup> We might loosely understand this as the set of possible artworks but it might also include other forms of human expression. The set contains all works that ever have been, will be, or could be created. Parallel conclusions would be reached if the space of works were modeled as continuous and appropriate modifications were made to the axioms.

Individuals from the community have standards as to which works in the set are obscene. A standard can be described by the subset of possible works which are considered to be obscene. Individual standards are assumed to be well-informed and made after deliberation and reflection. There is a single restriction on allowable standards: at least one work must be non-obscene.<sup>4</sup>

These individual standards are then aggregated to form a community standard. The community standard is subject to the same restriction as the individual standards: at least one work must be determined to be non-obscene. I place no other restrictions on the class of allowable standards. Individual standards and community standards are assumed to be subjective.

An aggregation rule is a systematic method of deriving the community standard from the individual standards.<sup>5</sup> Aggregation rules are studied through the axiomatic approach: several normative properties are formalized as axioms and the unique rule satisfying these axioms is characterized. I introduce four main axioms. Each is, in some way, a desirable property for any objective aggregation rule.

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<sup>2</sup>The model can also be applied to other types of legal standards, and has been used to study standards of behavior in the contexts of negligence (Miller and Perry, 2012), contract (Miller and Perry, 2013), and defamation law (Miller and Perry, forthcoming).

<sup>3</sup>Because each work may be thought of as a possible legal case, this model belongs to the family of case-space models introduced by Kornhauser (1992a) (see also Kornhauser, 1992b; Lax, 2007).

<sup>4</sup>Reasonable individuals should all believe that it is possible to create works which are non-obscene. Individuals who do not satisfy this restriction would be unreasonable as a matter of law. I do not require individuals to believe that some works must be obscene—there is no reason why individuals *must* be offended by anything.

<sup>5</sup>The aggregation rule only takes into account the individual standards about what is obscene. It does not incorporate the individuals’ higher order beliefs about what others consider to be obscene, except insofar as these beliefs affect their standards. An alternate approach might look at works which are commonly known to be obscene, or which are obscene according to a *p*-common belief (see Monderer and Samet, 1989).

The *homogeneity* axiom requires that if every member of the community shares a single standard, then that standard is also the community standard. If this axiom is not satisfied, then the community standard must be derived from something other than the individual standards.

The *responsiveness* axiom requires the community standard to “respond” in the same direction (more permissive or less) as the community. If every individual standard becomes more permissive, then the community standard should become more permissive as well. Responsiveness prevents the perverse result in which a defendant is convicted *because* the individuals in the community became more tolerant.

The *anonymity* axiom requires the aggregation rule to not discriminate between individuals. The law generally requires equal treatment of individuals. More specific to this case, the Supreme Court has held that the views of all adult members of the community must be taken into account in determining the community standard.<sup>6</sup>

The *neutrality*<sup>7</sup> axiom requires the aggregation rule to not discriminate, *ex ante*, between works. This axiom assumes that all standards are subjective and is relevant when there is no method by which works can be objectively compared. No court nor commentator has yet identified a plausible method of comparison. The lack of an objective method is largely what makes even personal views on obscenity difficult to define through a rule. A natural method to compare works would be to judge them by their parts; however, this is method was expressly disallowed by the Supreme Court.<sup>8</sup>

I show that any aggregation method which satisfies these four axioms must be “similar” to the unanimity rule, under which a work is deemed obscene when every individual considers it to be obscene, in the following sense: if every individual considers “enough” works to be permissible, then the outcome of the rule must coincide with the unanimity rule outcome.<sup>9</sup>

Monjardet (1990) and Nehring and Puppe (2007) show that the unanimity rule is the unique rule satisfying homogeneity, anonymity, neutrality, and *monotone independence*, an axiom which combines responsiveness with an independence property. I formulate two independence axioms and show that the responsiveness property can be removed if these axioms are assumed.

The U.S. Supreme Court has held that contemporary community standards are to be used in evaluating two elements of obscenity: (a) whether the work appeals to the prurient interest, and (b) whether the work is patently offensive.<sup>10</sup> This implies that individuals can, at least, make three types of judgments about the set of works that: (1) appeal to the prurient interest, (2) are patently offensive, and (3) are obscene; that is, which both appeal to the prurient interest and are patently offensive.

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<sup>6</sup>See *Roth v. United States*, 354 U.S. 476 (1957) and *Pinkus v. United States*, 436 U.S. 293 (1978).

<sup>7</sup>Neutrality is very different from the “neutrality” axiom of May (1952), who shows in a different context that anonymity, “neutrality,” and a stronger version of responsiveness imply majority rule.

<sup>8</sup>*Roth v. United States*, 354 U.S. 476 (1957).

<sup>9</sup>A formal definition of “enough” is provided in the main body of the paper.

<sup>10</sup>*Miller v. California*, 413 U.S. 15 (1973).

The first two types of judgments are not logically related. As a matter of law, a work may appeal to the prurient interest but not be patently offensive; alternatively, a work may be patently offensive but not appeal to the prurient interest. Were one judgment to imply the other, there would be no need for both elements to appear in the test. Each of the first two types of judgments, however, is clearly related to the third. If a work both appeals to the prurient interest and is patently offensive, then it is also obscene.

If there is a single community standard for obscenity, as has been assumed in this paper, then the judgments being aggregated are of the third type. We might label the resulting standard the ‘prurient interest and patently offensive’ community standard. However, one could infer from the Supreme Court opinions that there are two community standards, (a) the ‘prurient interest’ community standard and (b) the ‘patently offensive’ community standard.

A model of two community standards would take the following form. Individuals would make two separate judgments about which works (1) appeal to the prurient interest and (2) are patently offensive. The judgments would then be aggregated to form (a) the ‘prurient interest’ community standard and (b) the ‘patently offensive’ community standard.

This leads to two questions. First, if the two community standards are not aggregated independently—so that the individual judgments about which works are patently offensive could somehow be relevant in determining the ‘prurient interest’ community standard, and vice-versa—which aggregation rules would satisfy the axioms? The simple answer is that main result does not change in the case of two (or more) standards. Even if we allow for interdependent aggregation, any rule which satisfies the four axioms must be “similar” to unanimity rule.

Second, in the spirit of the literature on judgment aggregation (see Kornhauser and Sager, 1986; List and Pettit, 2002), we might ask what methods will allow us to independently aggregate the ‘prurient interest,’ ‘patently offensive,’ and ‘prurient interest and patently offensive’ standards, while preserving the logical relationship between the first two standards and the last. I show that in this case, if independence is assumed, the *unanimity rule* can be characterized with homogeneity and anonymity.<sup>11</sup>

Kasher and Rubinstein (1997) introduced a model of group identification in which individuals select the subset of the society whom they believe to be members of a group. While their model bears some formal similarity to the results in this paper, there are several significant differences. First, in their paper, each voter selected a subset of the voters, while in this paper there is no direct relationship between voters and alternatives. Ju (2010b) showed that the combination of the anonymity and neutrality axioms when the voters and alternatives are distinct can lead to very different results than the use of the seemingly similar symmetry axiom of Samet and Schmeidler (2003). Second, individuals in the Kasher-Rubinstein model were free to

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<sup>11</sup>This form of independence is referred to as *issue-independence* in Section 2.4.

choose any subset of the group, including the entire group and the empty set, while in this model individuals may not claim that all works are obscene. Third, the set of works is understood to be large, whereas the set of individuals in the Kasher-Rubinstein model was of size  $n$ , and thus Theorem 1 would be of little relevance in this setting.

There are two papers which study the group identification model with axiomatic frameworks similar to those employed in this paper. Samet and Schmeidler (2003) use a set of axioms similar to those employed in Theorem 2 to characterize the family of consent rules. Miller (2008) characterizes the class of ‘agreement rules’ (which includes unanimity rule) using *meet separability*, which is analogous to issue-independence in the domain considered in Section 2.5. For a general overview of the group identification literature, see Ju (2010a).

Kasher and Rubinstein (1997) also characterize a family of oligarchic rules, using a different model in which equivalence relations are aggregated. Relying on a result first proved by Mirkin (1975) (see also Leclerc, 1984; Barthélemy et al., 1986; Fishburn and Rubinstein, 1986), they show that any method of aggregating equivalence relations satisfying an independence condition and unanimity (analogous to the *homogeneity* axiom presented above) must be oligarchic. The *unanimity rule* is the only anonymous oligarchic rule. Because of the transitivity property of equivalence relations, neutrality is not needed in these results. More recent characterizations of oligarchic rules by Chambers and Miller (2011) and Dimitrov et al. (2012) are related to Theorem 3 in that they employ the meet separability property studied in Miller (2008).

## 2 The Model

### 2.1 Notation and the Model

The **community** is a set  $N \equiv \{1, \dots, n\}$  of individuals. There is a finite set of **works**, denoted by  $W$ . Let  $\mathcal{J} \equiv \{J \subseteq W : J \neq W\}$  be the set of **judgments**. The requirement that judgments be non-full subsets of  $W$  is a reasonableness condition that reflects the idea that not all works can be obscene, or should be prohibited. Let  $M \equiv \{1, \dots, m\}$  denote a finite set of **issues**. For example, if there is only a single standard of obscenity then  $m = 1$ , while if there is both a standard of ‘appeal to the prurient interest’ and ‘patently offensive’ then  $m = 2$ . A **standard** is an  $M$ -vector of judgments, one for each issue. The set of standards is denoted  $\mathcal{S} \equiv \mathcal{J}^M$ . A **profile** is an  $N$ -vector of standards,  $S = (S_1, \dots, S_n) \in \mathcal{S}^N$ , where  $S_i$  represent individual  $i$ ’s standard. I write  $S_{ij}$  to denote individual  $i$ ’s judgment about issue  $j$ . A rule  $f : \mathcal{S}^N \rightarrow \mathcal{S}$  is a function mapping each profile into a **community standard**, denoted  $f(S) = (f_1(S), \dots, f_m(S))$ .

For any finite set  $K$  and for any two sets  $S$  and  $T$  of the form  $\mathcal{J}^K$ , I define  $\sqcap$  as the coordinatewise intersection, so that  $(S \sqcap T)_k \equiv S_k \cap T_k$  for every  $k \in K$ , and I define  $\sqcup$  as the coordinatewise union, so that  $(S \sqcup T)_k \equiv S_k \cup T_k$ . Note that there

exist  $S, T \in \mathcal{J}^K$  such that  $S \sqcup T \notin \mathcal{J}^K$ . I define  $S \sqsubseteq T$  to mean that  $S_k \subseteq T_k$  for every  $k \in K$ . When  $S \sqsubseteq T$  I write that  $S$  is as **permissive** as  $T$ , because every work that a particular person permits in profile  $T$  is permitted by that person in profile  $S$ .<sup>12</sup> For a permutation  $\phi$  of  $W$ , I define  $(\phi S)_k \equiv \phi(S_k)$ .

## 2.2 Axioms

The first axiom, *homogeneity*, requires that if the community is perfectly homogeneous, so that every individual in the community has identical views about the entire standard, then this commonly held belief is the community standard.

**Homogeneity:** If  $S_i = S_j$  for all  $i, j \in N$ , then  $f(S) = S_1 = \dots = S_n$ .

Suppose that the individual standards change and that every individual's new standard is as permissive as was that individual's old standard (so that  $S_i \sqsubseteq S_i^*$  for all  $i \in N$ ). The second axiom, *responsiveness*, requires the resulting community standard to be as permissive as the prior community standard (so that  $f(S) \sqsubseteq f(S^*)$ ).

**Responsiveness:** If  $S \sqsubseteq S^*$ , then  $f(S) \sqsubseteq f(S^*)$ .

The principle of anonymity requires that each individual's view must be treated equally. Individuals' names are switched through a permutation  $\pi$  of  $N$ . For a given permutation,  $\pi(i)$  is the new name of the individual formerly known as  $i$ . For a given profile  $S$ ,  $\pi S \equiv (S_{\pi(1)}, \dots, S_{\pi(n)})$  is the profile that results once names are switched. The third axiom, *anonymity*, requires that permutations of the individuals' names do not affect the community standard.

**Anonymity:** For every permutation  $\pi$  of  $N$ ,  $f(S) = f(\pi S)$

The principle of *neutrality* is similar. Works' names are switched through a permutation  $\phi$  of  $W$ . For a given profile  $S$ ,  $f(\phi S)$  is the community standard derived from the profile that results when the names are switched; while  $\phi f(S)$  is the community standard that results when the names are switched only after the aggregation. The neutrality axiom requires that these two community standards be the same.

**Neutrality:** For every permutation  $\phi$  of  $W$ ,  $\phi(f(S)) = f(\phi S)$ .

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<sup>12</sup>Note that for  $K = 1$ , the symbols  $\sqcap$  and  $\sqcup$  are interchangeable, as are the symbols  $\sqcup$  and  $\sqcap$ , and the symbols  $\sqsubseteq$  and  $\supseteq$ .

## 2.3 The Unanimity Rule

Under the “unanimity rule,” a work is considered obscene if it is considered obscene by every individual. If there are multiple issues, then for each issue a work is prohibitible only when it is considered prohibitible by every individual.

**Unanimity Rule:** For every  $S \in \mathcal{S}^N$ ,  $f(S) = \bigcap_{i \in N} S_i$ .

The four axioms of homogeneity, responsiveness, anonymity, and neutrality are not by themselves sufficient to characterize the unanimity rule. The other rules that satisfy these axioms have a special property—their outcomes differ from the unanimity rule outcome only when individuals permit a very small number of works. These rules are all less permissive than the unanimity rule.

For example, consider the rule according to which a work A is determined to be obscene if (i) everyone permits exactly one work, one or more people consider A to be obscene, and some other work B is permitted by strictly more people than permit A, or (ii) at least one person permits two or more works, and everyone considers work A to be obscene. The outcome of this rule coincides with the unanimity rule outcome unless everyone permits exactly one work.

“Very small” refers to the cardinality of the set permitted works, and not a proportion; specifically, this cardinality must be  $m * n$ , where  $m$  is the number of issues and where  $n$  is the number of individuals in the community. To formalize this concept, let  $\mathcal{S}_{mn} = \{S \in \mathcal{S} : |W \setminus S_j| \geq m * n \text{ for all } j \in M\}$  be the set of standards in which each individual considers at least  $m * n$  works to be acceptable for each issue. I show that if each individual standard is in  $\mathcal{S}_{mn}$ , then the outcome of any rule which satisfies the four axioms must coincide with the unanimity rule outcome.

**Theorem 1.** *If an aggregation rule  $f$  satisfies homogeneity, responsiveness, anonymity, and neutrality, then for each  $S \in \mathcal{S}_{mn}^N$ ,  $f(S) = \bigcap_{i \in N} S_i$ .*

In the interpretation given to the set of works in the beginning, the set of works is finite but very large; under this interpretation one would expect all individual standards to be in  $\mathcal{S}_{mn}$ . Alternatively, were the set of works to be modeled as infinite, it would be natural to require the set of permitted works to be infinite (so that the set of permitted works would always be a positive proportion of the whole); in this case the individual standards would necessarily be in  $\mathcal{S}_{mn}$ , and unanimity rule would be the only rule satisfying the axioms.<sup>13</sup>

## 2.4 Independence

Monjardet (1990) and Nehring and Puppe (2007) prove a related result.<sup>14</sup> If there is a single issue ( $m = 1$ ), the unanimity rule is the unique rule satisfying homogeneity,

<sup>13</sup>We would need to modify *neutrality* for the infinite setting. For more, see Miller (2009).

<sup>14</sup>Several others, of course, have studied the aggregation of finite sets. For example, see Barberà et al. (1991) who study strategyproof voting rules under the domain of separable preferences.

anonymity, neutrality, and “monotone independence.”<sup>15</sup> The monotone independence axiom is a combination of responsiveness and “independence,” which states that whether a particular work is obscene depends only on the opinions about that work. In other words, opinions about *Adventures of Huckleberry Finn* can not be considered when determining whether *The Adventures of Tom Sawyer* is obscene.

Unanimity rule is clearly *independent* in the sense that the community standard’s judgment about a particular work given a particular issue depends only on the individual judgments about that work given that issue. This *independence* property can be broken into two strong axioms, *work-independence* and *issue-independence*. A rule is *work-independent* if the determination as to whether a particular work is obscene depends only on the opinions about that particular work. In the case of a single issue ( $m = 1$ ), monotone independence is equivalent to the combination of work-independence and responsiveness.

**Work-Independence:** If there exists  $w \in W$  and  $S, S' \in \mathcal{S}^N$  such that  $w \in S_{ij}$  if and only if  $w \in S'_{ij}$  for all  $i \in N$  and  $j \in M$ , then  $w \in f_j(S)$  if and only if  $w \in f_j(S')$ .

A rule is *issue-independent* if the collective judgment for each issue depends only on the opinions about that issue. This axiom is only meaningful when there are multiple issues ( $m = 2$ ).

**Issue-Independence:** If there exists  $j \in M$  and  $S, S' \in \mathcal{S}^N$  such that  $S_{ij} = S'_{ij}$  for all  $i \in N$ , then  $f_j(S) = f_j(S')$ .

The following characterization of the unanimity rule follows from Monjardet (1990) and Nehring and Puppe (2007).<sup>16</sup>

**Theorem 2.** *The unanimity rule is the only rule that satisfies homogeneity, anonymity, neutrality, work-independence, and issue-independence. Moreover, all five axioms are independent.*

In the legal interpretation of the model, responsiveness is desirable (because an increase in permissiveness should not lead to works being banned), while independence is not (beliefs about one work might be relevant in determining whether another work is obscene). In other settings, it is conceivable that independence will be more desirable than responsiveness. An implication of this theorem is that, in conjunction with the other axioms, the result will not change substantially; unanimity rule will be the implication if either responsiveness or the independence axioms are assumed.

<sup>15</sup>This follows the formulation of Nehring and Puppe (2007).

<sup>16</sup>Both Monjardet (1990) and Nehring and Puppe (2007) used “monotone independence” which includes responsiveness. However, as I show in the proof, responsiveness is implied by the other five axioms.



## 2.5 The Doctrinal Paradox

The doctrinal paradox of Kornhauser and Sager (1986) states that majority-rule aggregation of logically related issues can lead to inconsistent results. This idea and its subsequent formalization by List and Pettit (2002) has led to a significant amount of research on the aggregation of logically related issues.

While I have assumed that there is no logical relationship between the issues in  $M$ , there are cases when it would be natural to assume such a relationship. For example, if we were to include three issues, “appeal to the prurient interest,” “patent offensiveness,” and “obscenity,” we might think of the last issue as the conjunction of the previous two. A work is obscene only when it appeals to the prurient interest and is patently offensive.

To describe this formally, consider the model specified in Section 2.1, with the following changes. Let  $M^* \equiv \{a, b, a \wedge b\}$ , with the interpretation  $a$ =“appeals to the prurient interest,”  $b$ =“patently offensive,” and  $a \wedge b$  = “obscene”. Let  $\mathcal{S}^* \subseteq \mathcal{J}^{M^*}$  be the set of standards such that, for all  $S_i \in \mathcal{S}$  and  $S_{ia} \sqcap S_{ib} = S_{i(a \wedge b)}$ . Let  $f^* : \mathcal{S}^{*N} \rightarrow \mathcal{S}^*$  denote a rule in this domain.

If we add an additional assumption of issue-independence, this formal setup allows us to remove two unnecessary axioms: responsiveness and neutrality. The combination of the issue-independence, homogeneity, and anonymity axioms is sufficient to characterize the unanimity rule. This result is related to that of Ahn and Chambers (2010), who characterize the unanimity rule using homogeneity, responsiveness, anonymity, and disjoint additivity.<sup>17</sup>

**Theorem 3.** *An aggregation rule  $f^*$  satisfies homogeneity, anonymity, and issue-independence if and only if it is unanimity rule. Furthermore, the three axioms are independent.*

## Appendix

### Proof of Theorem 1

Let  $f$  satisfy the four axioms.

**Step 1.** I show that for any profile  $S \in \mathcal{S}^N$ ,  $\sqcap_{i \in N} S_i \sqsubseteq f(S)$ . Let  $S \in \mathcal{S}^N$ . Let  $S' \equiv (\sqcap_{i \in N} S_i)^N$ , the  $N$ -vector for which each element is  $\sqcap_{i \in N} S_i$ . Clearly,  $S' \sqsubseteq S$ . By homogeneity,  $f(S') = \sqcap_{i \in N} S_i$ . Responsiveness implies that  $\sqcap_{i \in N} S_i \sqsubseteq f(S)$ .

**Step 2:** I show that if there is a profile  $T \in \mathcal{S}^N$  such that (a)  $T_{ik} \cup T_{jl} = W$  unless  $i = j$  and  $k = l$ , and (b)  $|T_{ik}| = |T_{jk}|$  for all  $i, j \in N$  and  $k \in M$ , then  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ .

Let  $T \in \mathcal{S}^N$  be such that conditions (a) and (b) are met. Without loss of generality, let  $w \notin T_{11}$ . To prove that  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ , it is sufficient to show that  $w \notin f_1(T)$ .

<sup>17</sup>Issue-independence and homogeneity together imply responsiveness and disjoint additivity.

Suppose, contrariwise, that  $w \in f_1(T)$ . Then, by neutrality,  $W \setminus T_{11} \sqsubseteq f_1(T)$ . By anonymity and neutrality,  $W \setminus T_{i1} \sqsubseteq f_1(T)$  for all  $i \in N$ . Thus  $\sqcup_{i \in N} (W \setminus T_{i1}) \sqsubseteq f_1(T)$ . By step 1,  $\sqcap_{i \in N} T_{i1} \sqsubseteq f_1(T)$ , which implies that  $f_1(T) = W$ . But this is a contradiction, which proves that  $w \notin f_1(T)$ , and therefore that  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ .

**Step 3:** I show that  $f(S) \sqsubseteq \sqcap_{i \in N} S_i$  for all  $S \in \mathcal{S}_{mn}^N$ . Let  $S \in \mathcal{S}_{mn}^N$  and let  $w \notin S_{11}$ . To show that  $f(S) \sqsubseteq \sqcap_{i \in N} S_i$ , it is sufficient to show that  $w \notin f_1(S)$ . Let  $S^* \in \mathcal{S}^N$  be a profile such that (a)  $w \notin S_{11}^*$ , (b)  $S_{ik}^* \cup S_{jl}^* = W$  unless  $i = j$  and  $k = l$ , (c)  $|W \setminus S_{ij}^*| = 1$  for all  $i \in N$  and  $j \in M$ , and (d)  $S \sqsubseteq S^*$ . Note that such a profile  $S^*$  is guaranteed to exist for all  $S \in \mathcal{S}_{mn}^N$ . By step 2,  $f(S^*) \sqsubseteq \sqcap_{i \in N} S_i^*$ . Because  $S \sqsubseteq S^*$ , responsiveness implies that  $f_1(S) \sqsubseteq f_1(S^*)$ , and therefore  $w \notin f_1(S)$ .

**Step 4:** Steps 1 and 3 directly imply that  $f(S) = \sqcap_{i \in N} S_i$  for all  $S \in \mathcal{S}_{mn}^N$ .

## Proof of Theorem 2

That unanimity rule satisfies the five axioms is trivial. I will show that any rule which satisfies the five axioms must be unanimity rule. Let  $f$  satisfy the five axioms.

Issue-independence and work-independence imply that, for each issue  $j \in M$  and each work  $w \in W$ , there exists a group of coalitions  $G_{jw} \subseteq 2^N$  such that  $w \in f_j(S)$  if and only if  $\{i \in N : w \in S_{ij}\} \in G_{jw}$ . Neutrality implies that there exists a single such group of coalitions  $G_j$  for each issue  $j$  such that  $G_j = G_{jw}$  for all  $w \in W$ . Anonymity implies that there is a collection of quotas,  $Q_j \subseteq \{0, \dots, n\}$ , such that  $w \in f_j(S)$  if and only if  $|\{i \in N : w \in S_{ij}\}| \in Q_j$ . Homogeneity implies that  $Q_j \neq \emptyset$ .

Let  $j \in M$ , let  $x \in \{0, \dots, n-1\}$ , and let  $S \in \mathcal{S}^N$  such that, for all  $w \in W$ ,  $|\{i \in N : w \in S_{ij}\}| = x$ . Then  $f_j(S) = W$  if  $x \in Q_j$  and  $f_j(S) = \emptyset$ , otherwise. Clearly  $f_j(S) \neq W$  and therefore  $\{0, \dots, n-1\} \not\subseteq Q_j$ . Because  $Q_j \neq \emptyset$  it follows that  $Q_j = \{n\}$  and therefore  $f(S) = \sqcap_{i \in N} S_i$ .

For proof that the axioms are independent, see Miller (2009).

## Proof of Theorem 3

That unanimity rule satisfies the three axioms is trivial. To prove the converse, let  $f^*$  satisfy the three axioms. I will show that  $f^*$  must be unanimity rule.

Issue-independence implies that there are functions  $g_a, g_b, g_{a \wedge b} : \mathcal{J}^N \rightarrow \mathcal{J}$  such that, for all  $S \in \mathcal{S}^{*N}$ ,  $f^*(S) = \left( g_a \left( (S_{ia})_{i \in N} \right), g_b \left( (S_{ib})_{i \in N} \right), g_{a \wedge b} \left( (S_{i(a \wedge b)})_{i \in N} \right) \right)$  such that, for all  $x, y \in \mathcal{J}^N$ ,  $g_a(x) \sqcap g_b(y) = g_{a \wedge b}(x \sqcap y)$ . Furthermore,  $g_{a \wedge b}(x)$  must be responsive. To see why, assume that  $x \sqsubseteq z$ . Clearly,  $g_a(x) \sqcap g_b(z) = g_{a \wedge b}(x) = g_a(z) \sqcap g_b(x)$ . This implies that  $g_{a \wedge b}(x) \sqsubseteq g_a(z) \sqcap g_b(z)$  and therefore  $g_{a \wedge b}(x) \sqsubseteq g_{a \wedge b}(z)$ .

Homogeneity implies that, for all  $x \in \mathcal{J}^N$ ,  $g_a(x) = g_b(x) = g_{a \wedge b}(x)$ . To see why, suppose, contrariwise, that there is an  $x \in \mathcal{J}^N$  such that  $g_a(x) \neq g_b(x)$ . We know that  $g_a(x) \sqcap g_b(x) = g_{a \wedge b}(x)$ . This implies that either  $g_a(x) \supsetneq g_{a \wedge b}(x)$  or  $g_b(x) \supsetneq g_{a \wedge b}(x)$  or both. Without loss of generality, assume that  $g_a(x) \supsetneq g_{a \wedge b}(x)$ . For all  $z \in \mathcal{J}^N$ ,  $g_a(x) \sqcap g_b(z) = g_{a \wedge b}(x \sqcap z)$ . Let  $z \equiv (g_a(x))^N$ , the  $N$ -vector for which

every element is equal to  $g_a(x)$ . By homogeneity,  $g_b(z) = g_a(x)$  which implies that  $g_a(x) \sqcap g_a(x) = g_a(x) = g_{a \wedge b}(x \sqcap z)$ . But because  $g_{a \wedge b}(x) \supseteq g_{a \wedge b}(x \sqcap z)$ , this violates the assumption that  $g_a(x) \supsetneq g_{a \wedge b}(x)$  and proves that, for all  $x \in \mathcal{J}^N$ ,  $g_a(x) = g_b(x)$ . Therefore,  $g_a(x) = g_{a \wedge b}(x)$ . Let  $g(x) \equiv g_a(x)$ .

Let  $x \in \mathcal{J}^N$ , and let  $\pi$  be the permutation such that  $\pi(n) = 1$  and, for all  $i < n$ ,  $\pi(i) = i + 1$ . By anonymity,  $g(x) = g(\pi x)$ . It follows that  $g(x) = g(x) \sqcap g(\pi x) = g(x \sqcap \pi x)$ . By induction, this implies that  $g(x) = g(x \sqcap \pi x \sqcap \pi \pi x \sqcap \dots) = g(\sqcap_{i \in N} x_i, \dots, \sqcap_{i \in N} x_i)$ . From homogeneity it follows that  $g(x) = \sqcap_{i \in N} x_i$  which implies that for all  $S \in \mathcal{S}^{*N}$ ,  $f^*(S) = (\sqcap_{i \in N} S_{ia}, \sqcap_{i \in N} S_{ib}, \sqcap_{i \in N} S_{i(a \wedge b)}) = \sqcap_{i \in N} S_i$ .

For proof that the axioms are independent, see Miller (2009).

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