Supplementary Notes to Open Tables

Full Proofs

There are a few basic equalities that will be used throughout the proofs. First, let $\beta = (\beta_0, \beta_1, ..., \beta_n)$ be an arbitrary $n+1$ dimensional vector. Then:

$$\sum_{i=0}^{n} \binom{n}{i} (1-F(x))^i F(x)^{n-i} \beta_i = \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i} \binom{n-m}{j} (1-F(x))^{i+j} F(x)^{n-i-j} \beta_{i+j} \quad (3)$$

Proof. By Vandermonde's identity, $\binom{n}{i} = \sum_{j=0}^{i} \binom{m}{i-j} \binom{n-m}{j}$.

Thus the left hand side of (3) is equal to $\sum_{i=0}^{n} \sum_{j=0}^{i} \binom{m}{i-j} \binom{n-m}{j} (1-F(x))^i F(x)^{n-i} \beta_i$.

If we replace i with $i+j$, then this becomes $\sum_{i+j=0}^{n} \sum_{j=0}^{i+j} \binom{m}{i} \binom{n-m}{j} (1-F(x))^{i+j} F(x)^{n-i-j} \beta_{i+j}$.

We can rewrite this expression as $\sum_{i=0}^{n} \sum_{j=0}^{n-i} \binom{m}{i} \binom{n-m}{j} (1-F(x))^{i+j} F(x)^{n-i-j} \beta_{i+j}$.

Because $\binom{a}{b} = 0$ for $a < b$, this is equivalent to $\sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i} \binom{n-m}{j} (1-F(x))^{i+j} F(x)^{n-i-j} \beta_{i+j}$. □

$$\kappa(x) = \lambda(x,x) + m(1-F(x)) \quad (4)$$

Proof. Recall that $\kappa(x) = \sum_{i=0}^{n} \binom{n}{i} (1-F(x))^i F(x)^{n-i} \min\{m, i\}$.

By expression (3), this equals $\sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i} \binom{n-m}{j} (1-F(x))^{i+j} F(x)^{n-i-j} \min\{m, i+j\}$.

Rearranging terms, thus becomes:

$$\sum_{i=0}^{m} \binom{m}{i} (1-F(x))^i F(x)^{m-i} \sum_{j=0}^{n-m} \binom{n-m}{j} (1-F(x))^j F(x)^{n-m-j} (\min\{m-i, j\} + i).$$

This last expression is equivalent to $\lambda(x,x) + m(1-F(x))$. □

$$\frac{d}{dx} E[v|v \geq x] = \frac{f(x)}{1-F(x)} (E[v|v \geq x] - x) \quad (5)$$

Proof. Recall that $E[v|v \geq x] = \int_x^\infty \frac{vf(v)}{1-F(x)} dv$, or $\frac{1}{1-F(x)} \int_x^\infty vf(v)dv$. Using the chain rule,

$\frac{d}{dx}E\left[v|v\geq x\right]=\frac{f(x)}{(1-F(x))^2}\int_x^\infty vf(v)dv-\frac{1}{1-F(x)}xf(x)$, or $\frac{f(x)}{1-F(x)}\left(E\left[v|v\geq x\right]-x\right)$.

□

The proofs of the following thre statements are straightforward and left to readers.

$$\kappa'(x)=\sum_{i=0}^{n}\binom{n}{i}\left(1-F(x)\right)^{i}F(x)^{n-i}\left(\frac{n-i}{F(x)}-\frac{i}{1-F(x)}\right)f(x)\min\left\{m,i\right\} \tag{6}$$

$$\left.\frac{dv^*}{dc}\right|_{c=0}=\frac{n\left(1-F(p)\right)}{\kappa(p)} \tag{7}$$

$$\left.\frac{d\hat{v}}{dc}\right|_{c=0}=\frac{\left(n-m\right)\left(1-F(p)\right)}{\lambda(p,p)} \tag{8}$$

Proof of Theorems 2.1 and 2.2.

If we take the derivative of $W_r(c)$ with respect to the transportation cost, c, we get:

$$W_r'(c)=\kappa'(p+c)\left(E\left[v|v\geq p+c\right]-c\right)+\kappa(p+c)\left(\left[\frac{d}{d(p+c)}E\left[v|v\geq p+c\right]\right]-1\right).$$

Evaluated at $c=0$, this becomes:

$$W_r'(0)=\kappa'(p)E\left[v|v\geq p\right]+\kappa(p)\left[\frac{d}{dp}E\left[v|v\geq p\right]\right]-\kappa(p).$$

If we take the derivative of $W_o(c)$ with respect to the transportation cost, c, we get:

$$W_o'(c)=\kappa'(v^*)\frac{dv^*}{dc}E\left[v|v\geq v^*\right]+\kappa(v^*)\left[\frac{d}{dv^*}E\left[v|v\geq v^*\right]\right]\frac{dv^*}{dc}-n\left(1-F(v^*)\right)+nf(v^*)c\frac{dv^*}{dc}.$$

Evaluated at $c=0$, this becomes:

$$W_o'(0)=\kappa'(p)\left.\frac{dv^*}{dc}\right|_{c=0}E\left[v|v\geq p\right]+\kappa(p)\left[\frac{d}{dp}E\left[v|v\geq p\right]\right]\left.\frac{dv^*}{dc}\right|_{c=0}-n\left(1-F(v^*)\right).$$

Using expression (7) and simplifying, we have:

$$W_o'(0)=\left(\kappa'(p)E\left[v|v\geq p\right]+\kappa(p)\left[\frac{d}{dp}E\left[v|v\geq p\right]\right]-\kappa(p)\right)\left.\frac{dv^*}{dc}\right|_{c=0}.$$

Or, $W_o'(0)=W_r'(0)\left.\frac{dv^*}{dc}\right|_{c=0}$. Expression (7) is greater than one, thus

$W_r'(0)\geq W_o'(0)$ if and only if $W_r'(0)\leq 0$.

This is equivalent to: $\kappa(p)\frac{f(p)}{1-F(p)}\left(E\left[v|v \geq p\right] - p\right) \leq \kappa(p) - \kappa'(p)E\left[v|v \geq p\right]$.

Evaluated at $p = 0$ this is: $\kappa(0)f(0)E\left[v|v \geq 0\right] \leq \kappa(0)$.

This is true if and only if: $f(0)\int_0^\infty xf(x)dx \leq 1$, which proves Theorem 2.1.

By assumption, $\Gamma'(p) \leq 0$, which implies that $\frac{f(p)}{1-F(p)}\left(E\left[v|v \geq p\right] - p\right) \leq 1$.

This implies that $\kappa(p)\frac{f(p)}{1-F(p)}\left(E\left[v|v \geq p\right] - p\right) \leq \kappa(p)$.

It follows from the fact that $\kappa'(p) \leq 0$ and $E\left[v|v \geq p\right] > 0$ that $W_r'(0) \geq W_o'(0)$ for all prices p.

Furthermore, because $\kappa'(p) < 0$ for all $p > 0$, it follows that $W_r'(0) > W_o'(0)$ for all prices $p > 0$.

At $p = 0$, the fact that $\kappa'(0) = 0$ implies that $W_r'(0) > W_o'(0)$ if and only if

$\kappa(p)\frac{f(p)}{1-F(p)}\left(E\left[v|v \geq p\right] - p\right) < \kappa(p)$, which is true if and only if $\Gamma'(0) < 0$.

This proves Theorem 2.2.

Proof of Theorem 2.3.

At $c = 0$, $W_o(c,p) = W_r(c,p)$ so $\arg\max_p W_o(c,p) = \arg\max_p W_r(c,p)$. Thus $W_o(0,p_o) = W_r(0,p_r)$. To evaluate whether reservations dominates open tables we compare the first derivatives with respect to c.

$$\frac{d}{dc}W_o(c,p_o) = \kappa'\left(v^*\right)\frac{dv^*}{dc}E\left[v|v \geq v^*\right] + \kappa\left(v^*\right)\frac{d}{dv^*}E\left[v|v \geq v^*\right]\frac{dv^*}{dc} + n*c*f\left(v^*\right)\frac{dv^*}{dc} - n\left(1 - F\left(v^*\right)\right),$$

or

$$\left(\kappa'\left(v^*\right)E\left[v|v \geq v^*\right] + \kappa\left(v^*\right)\frac{d}{dv^*}E\left[v|v \geq v^*\right] + n*c*f\left(v^*\right)\right)\frac{dv^*}{dc} - n\left(1 - F\left(v^*\right)\right).$$

Because $p_o = \arg\max_p W_o(c,p)$, it follows that $\frac{d}{dp}W_o(c,p_o) = 0$.

$$\frac{d}{dp}W_o(c,p) = \kappa'\left(v^*\right)\frac{dv^*}{dp}E\left[v|v \geq v^*\right] + \kappa\left(v^*\right)\frac{d}{dv^*}E\left[v|v \geq v^*\right]\frac{dv^*}{dp} + n*c*f\left(v^*\right)\frac{dv^*}{dp}.$$

Because $\frac{dv^*}{dp} > 0$, this implies that

$$\kappa'(v^*) E[v|v \geq v^*] + \kappa(v^*) \frac{d}{dv^*} E[v|v \geq v^*] + n * c * f(v^*) = 0.$$

Therefore, $\frac{d}{dc} W_o(c, p_o) = -n(1 - F(v^*))$.
If we let c go to zero, $\frac{d}{dc} W_o(0, p_o) = -n(1 - F(p))$, where $p \equiv \lim_{c \to 0} p_o = \lim_{c \to 0} p_r$.

Next, $\frac{d}{dc} W_r(c, p_r) = \kappa'(p + c)(E[v|v \geq p + c] - c) + \kappa(p + c)\left(\frac{d}{d(p+c)} E[v|v \geq v^*] - 1\right)$.
Recall that $p_r = \arg\max_p W_r(c, p)$. Consequently, $\frac{d}{dp} W_r(c, p_r) = 0$.
Because $\frac{d}{dp} W_r(c, p) = \kappa'(p + c)(E[v|v \geq p + c] - c) + \kappa(p + c) \frac{d}{d(p+c)} E[v|v \geq v^*]$, it follows that $\frac{d}{dc} W_r(c, p_r) = -\kappa(p + c)$. If we let c go to zero, $\frac{d}{dc} W_r(0, p_r) = -\kappa(p)$. Because $-\kappa(p) > -n(1 - F(p))$, it follows that, at sufficiently small c, $W_r(c, p_o) > W_o(c, p_r)$.

Unstated Theorems

The following two theorems and lemma are alluded to in the text:

Theorem 3.1. *Suppose that the price* $p_o = \arg\max_p p * \kappa(v^*)$ *and that the price* $p_r = \arg\max_p p * \kappa(p + c)$. *For sufficiently small* c, *if* $\Gamma'(p) \leq 0$, *then* $W_r(c, p_r) > W_o(c, p_o)$.

Theorem 3.2. *Suppose that the price* $p_o = \arg\max_p p * \kappa(v^*)$ *and that the price* $p_r = \arg\max_p p * \kappa(p + c)$. *For sufficiently small* c, *if* $W_o(c, p_o) > W_r(c, p_r)$, *then* $p_o < p^*$, *where* p^* *is the socially optimal price.*

Proof. At $c = 0$, $v^* = p + c = p$, so $\arg\max_p p * \kappa(v^*)|_{c=0} = \arg\max_p p * \kappa(p + c)|_{c=0}$. Thus $W_o(0, p_o) = W_r(0, p_r)$. To evaluate whether reservations dominates open tables we compare the first derivatives with respect to c.

$$\frac{d}{dc} W_o(c, p) = \kappa'(v^*) \frac{dv^*}{dc} E[v|v \geq v^*] + \kappa(v^*) \frac{d}{dv^*} E[v|v \geq v^*] \frac{dv^*}{dc} + n*c*f(v^*) \frac{dv^*}{dc} - n(1 - F(v^*)),$$

or

$$\left(\kappa'(v^*) E[v|v \geq v^*] + \kappa(v^*) \frac{d}{dv^*} E[v|v \geq v^*] + n * c * f(v^*) - \kappa((v^*)\right) \frac{dv^*}{dc}.$$

The profit maximizing price $p_o = \arg\max_p p*\kappa(v^*)$ is given by $\kappa(v^*) = -p_o \kappa'(v^*) \left.\frac{dv^*}{dp}\right|_{p=p_o}$, and therefore

$$\frac{d}{dc} W_o(c, p_o) = \left(\kappa'(v^*) E[v|v \geq v^*] - p_o \kappa'(v^*) \left.\frac{dv^*}{dp}\right|_{p=p_o} \Gamma'(v^*) + n * c * f(v^*)\right) \frac{dv^*}{dc}.$$

Because $\lim_{c\to 0} \frac{dv^*}{dp}\Big|_{p=p_o} = 0$, it follows that

$$\frac{d}{dc}W_o(0,p_o) = \kappa'(p)\left(E\left[v|v \geq p\right] - p\Gamma'(p)\right)\frac{dv^*}{dc}\Big|_{c=0}.$$

Next, $\frac{d}{dc}W_r(c,p) = \kappa'(p+c)\left(E\left[v|v \geq p+c\right] - c\right) + \kappa(p+c)\left(\frac{d}{d(p+c)}E\left[v|v \geq p+c\right] - 1\right)$.
The profit maximizing price $p_r = \arg\max_p p{*}\kappa(p+c)$ is given by $\kappa(p_r+c) = -p_r\kappa'(p_r+c)$, and thus:

$$\frac{d}{dc}W_r(c,p_r) = \kappa'(p_r+c)\left(E\left[v|v \geq p_r+c\right] - c\right) - p_r\kappa'(p_r+c)\,\Gamma'(p+c)$$

If we let c go to zero, $\frac{d}{dc}W_r(0,p_r) = \kappa'(p)\left(E\left[v|v \geq p\right] - p\Gamma'(p)\right)$.
Therefore, $\frac{d}{dc}W_o(0,p_o) = \frac{d}{dc}W_r(0,p_r) * \frac{dv^*}{dc}\Big|_{c=0}$.
As $\frac{dv^*}{dc}\Big|_{c=0} > 1$, it follows that $\frac{d}{dc}W_o(0,p_o) \geq \frac{d}{dc}W_r(0,p_r)$ if and only if $\frac{d}{dc}W_r(0,p_r) > 0$. Therefore, social welfare is increasing at the profit-maximizing price. This proves Theorem 2.5. Because $\kappa'(p) < 0$, it follows that $\frac{d}{dc}W_o(0,p_o) \geq \frac{d}{dc}W_r(0,p_r)$ if and only if $E\left[v|v \geq p\right] \leq p\Gamma'(p)$. By assumption $\Gamma'(p) \leq 0$, therefore $\frac{d}{dc}W_r(0,p_r) \geq \frac{d}{dc}W_o(0,p_o)$. This proves Theorem 2.4. □

Lemma 3.3. *Suppose that the price* $p_o = \arg\max_p p * \kappa(v^*)$ *and that the price* $p_r = \arg\max_p p * \kappa(p+c)$. *For any* c, $p_o * \kappa(v^*) < p_r * \kappa(p_r+c)$.

Proof. The function $\kappa(p)$ is decreasing in p. Because $v^* \geq p+c$, it follows that, for any price p, $p{*}\kappa(v^*) \leq p{*}\kappa(p+c)$. Therefore, $p_o{*}\kappa\left(v^*|_{c=0}\right) < p_o{*}\kappa(p_o+c)$. By construction, because p_r is the profit-maximizing price, $p_o * \kappa(p_o+c) < p_r * \kappa(p_r+c)$. Therefore $p_o * \kappa(v^*) < p_r * \kappa(p_r+c)$. □

Proof of Lemma 2.4.

If $c = 0$, $\hat{v} = p$, and thus $W_s(0) = m\left(1 - F(p)\right)E[v|v \geq p] + \lambda(p,p)E[v|v \geq p]$.

By expression (4), this equals $\kappa(p)E[v|v \geq p] = W_o(0)$.

Proof of Lemma 2.5.

Let $O(c) = \kappa(v^*)$, and let $S(c) = m(1 - F(p+c)) + \lambda(\hat{v}, p+c)$.
When $c = 0$, $v^* = \hat{v} = p$, therefore, $O(0) = \kappa(p)$ and $S(0) = m(1 - F(p)) + \lambda(p,p)$. By statement (4), $\kappa(p) = m(1 - F(p)) + \lambda(p,p)$, and therefore $O(0) = S(0)$.
Computing the derivatives, we find that $O'(c) = \kappa'(v^*)\,\frac{dv^*}{dc}\Big|_{c=0}$ and therefore, $O'(0) = \frac{n(1-F(p))\kappa'(p)}{\kappa(p)}$. Also, $S'(c) = \frac{d}{dc}\lambda(\hat{v}, p+c) - mf(p+c)$, and therefore $S'(0) = \frac{d}{dc}\lambda(\hat{v}, p+c)\Big|_{c=0} -$

$mf(p)$. Therefore, $O'(0) \geq S'(0)$ if and only if

$$\frac{n\,(1-F(p))\,\kappa'(p)}{\kappa(p)} \geq \left.\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\right|_{c=0} - mf(p). \tag{9}$$

Note that $\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\} =$
$-\frac{f(p+c)}{(1-F(p+c))F(p+c)} \sum_{i=0}^{m} \binom{m}{i} (1-F(p+c))^{i}\, F(p{+}c)^{m-i} \sum_{j=0}^{n-m} \binom{n-m}{j} (1-F(\hat{v}))^{j}\, F(\hat{v})^{n-m-j}$
$\min\{m-i, j\} i + \frac{mf(p+c)}{F(p+c)}\lambda(\hat{v}, p+c)$
$-\frac{f(\hat{v})}{(1-F(\hat{v}))F(\hat{v})} \frac{d\hat{v}}{dc} \sum_{i=0}^{m} \binom{m}{i} (1-F(p+c))^{i}\, F(p+c)^{m-i} \sum_{j=0}^{n-m} \binom{n-m}{j} (1-F(\hat{v}))^{j}\, F(\hat{v})^{n-m-j}$
$\min\{m-i, j\} j + \frac{(n-m)f(\hat{v})}{F(\hat{v})} \frac{d\hat{v}}{dc}\lambda(\hat{v}, p+c)$.
Evaluated at $c = 0$, this becomes: $\left.\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\right|_{c=0} =$
$-\frac{f(p)}{(1-F(p))F(p)} \sum_{i=0}^{m} \binom{m}{i} \sum_{j=0}^{n-m} (1-F(p))^{i+j}\, F(p)^{n-i-j} \min\{m-i, j\} i + \frac{mf(p)}{F(p)}\lambda(p, p)$
$-\frac{f(p)}{(1-F(p))F(p)} \left.\frac{d\hat{v}}{dc}\right|_{c=0} \sum_{i=0}^{m} \binom{m}{i} \sum_{j=0}^{n-m} (1-F(p))^{i+j}\, F(p)^{n-i-j} \min\{m{-}i, j\} j {+} \frac{(n-m)f(p)}{F(p)} \left.\frac{d\hat{v}}{dc}\right|_{c=0} \lambda(p, p)$.
Using expression (4), and collecting terms, this simplifes to:
$\left.\frac{d}{dc}\lambda\left(\hat{v}, p+c\right)\right|_{c=0} = \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)} \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right) \left(\kappa(p) - m\,(1-F(p))\right)$
$-\frac{f(p)}{(1-F(p))F(p)} \sum_{i=0}^{m} \binom{m}{i} \sum_{j=0}^{n-m} (1-F(p))^{i+j}\, F(p)^{n-i-j} \min\{m-i, j\} \left(i + j \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right)$.
Thus expression (10) becomes:
$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} + mf(p) - \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)} \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right) \left(\kappa(p) - m\,(1-F(p))\right) \geq$
$-\frac{f(p)}{(1-F(p))F(p)} \sum_{i=0}^{m} \binom{m}{i} \sum_{j=0}^{n-m} (1-F(p))^{i+j}\, F(p)^{n-i-j} \min\{m-i, j\} \left(i + j \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right)$.
Or:
$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} {+} \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)} \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right) (m - \kappa(p)) {-} mf(p) \left(m + (n-m) \left.\frac{d\hat{v}}{dc}\right|_{c=0} - 1\right) \geq$
$-\frac{f(p)}{(1-F(p))F(p)} \sum_{i=0}^{m} \binom{m}{i} \sum_{j=0}^{n-m} (1-F(p))^{i+j}\, F(p)^{n-i-j} \min\{m-i, j\} \left(i + j \left.\frac{d\hat{v}}{dc}\right|_{c=0}\right)$.
Evaluated at $c = 0$, and using expression (8), this becomes:

$$\left.\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\right|_{c=0} = f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} (1-F(p))^{i+j}\, F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) + \left(\frac{n-m-j}{F(p)} - \frac{j}{1-F(p)}\right) \frac{(n-m)(1-F(p))}{\lambda(p,p)}\right) \min\{m-i, j\}$$

This simplifies to:

$$\left.\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\right|_{c=0} = f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} (1-F(p))^{i+j}\, F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) + \left(\frac{n-m-j}{F(p)} - \frac{j}{1-F(p)}\right) \frac{(n-m)(1-F(p))}{\lambda(p,p)}\right) \min\{m-i, j\}$$

Thus $W_o'(0) \geq W_s'(0)$ if and only if:

$$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} \geq f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} (1-F(p))^{i+j}\, F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) + \left(\frac{n-m-j}{F(p)} - \frac{j}{1-F(p)}\right) \frac{(n-m)(1-F(p))}{\lambda(p,p)}\right) \min\{m-i,j\} - mf(p)$$

Multiplying each side by $\lambda(p,p)\kappa(p)$:

$\lambda(p,p)n\left(1-F(p)\right)\kappa'(p) \geq f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \min\{m-i,j\}$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) \lambda(p,p)\kappa(p) + \left(\frac{n-m-j}{F(p)} - \frac{j+\frac{m}{n-m}}{1-F(p)}\right) \kappa(p)(n-m)\left(1-F(p)\right)\right)$$

Combining statements (6) and (3):

$$\kappa'(p) = \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \left(\frac{n-i-j}{F(p)} - \frac{i+j}{1-F(p)}\right) f(p) \min\{m, i+j\}.$$

Rearranging terms and applying statement (3):

$$\kappa'(p) = f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \left(\frac{n-i-j}{F(p)} - \frac{i+j}{1-F(p)}\right) \min\{m-i,j\} +$$

$$mf(p)\left(1-F(p)\right) \sum_{i=0}^{n-1} \binom{n-1}{i} \left(1-F(p)\right)^{i} F(p)^{n-1-i} \left(\frac{n-1-i}{F(p)} - \frac{i+1}{1-F(p)}\right).$$

Note that $\sum_{i=0}^{n-1} \binom{n-1}{i} \left(1-F(p)\right)^{i} F(p)^{n-1-i} \left(\frac{n-1-i}{F(p)} - \frac{i+1}{1-F(p)}\right) = \frac{-1}{1-F(p)}$. Thus:

$$\kappa'(p) = f(p) \sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \left(\frac{n-i-j}{F(p)} - \frac{i+j}{1-F(p)}\right) \min\{m-i,j\} - mf(p).$$

Substituting for $\kappa'(p)$ and dividing each side by $f(p)$, it follows that $W_o'(p) \geq W_s'(p)$ if and only if:

$$\sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \min\{m-i,j\}$$

$$\left(\left(\frac{n-i-j}{F(p)} - \frac{i+j}{1-F(p)}\right) \lambda(p,p) - m\right) n\left(1-F(p)\right) \geq$$

$$\sum_{i=0}^{m} \sum_{j=0}^{n-m} \binom{m}{i}\binom{n-m}{j} \left(1-F(p)\right)^{i+j} F(p)^{n-i-j} \min\{m-i,j\}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) \lambda(p,p)\kappa(p) + \left(\frac{n-m-j}{F(p)} - \frac{j+\frac{m}{n-m}}{1-F(p)}\right) \kappa(p)(n-m)\left(1-F(p)\right)\right)$$

Multiplying each side by $F(p)$:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n^2(1-F(p))\lambda(p,p)-n(i+j)\lambda(p,p)-mn(1-F(p))F(p)\right)\geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(m\lambda(p,p)-\frac{i\lambda(p,p)}{1-F(p)}+(n-m)^2(1-F(p))-(n-m)j-mF(p)\right)\kappa(p)$$

Rearranging terms:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n^2(1-F(p))\lambda(p,p)-mn(1-F(p))F(p)+\left(mF(p)-m\lambda(p,p)-(n-m)^2(1-F(p))\right)\kappa(p)\right)\geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n(i+j)\lambda(p,p)-\frac{i\lambda(p,p)\kappa(p)}{1-F(p)}-(n-m)j\kappa(p)\right)$$

Using the substitution in statement (4) and cancelling terms:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left[-m\left(n(1-F(p))-\kappa(p)\right)^2-mF(p)\left(n(1-F(p))-\kappa(p)\right)\right]\geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n(i+j)\left(\kappa(p)-m(1-F(p))\right)-\frac{i(\kappa(p)-m(1-F(p)))\kappa(p)}{1-F(p)}-(n-m)j\kappa(p)\right)$$

Note that $\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}X_{ij}$

$$=\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}(m-i)X_{ij}$$

$$-\sum_{i=0}^{m}\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}(m-i-j)X_{ij}.$$

It follows that $W_o'(p)\geq W_s'(p)$ if and only if:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^m\binom{n}{i}\left(1-F(p)\right)^iF(p)^{n-i}(m-i)\right]\geq$$

$$m\sum_{i=0}^m\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-\sum_{i=0}^m\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}i$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-\sum_{i=0}^m\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)(i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)+$$

$$\sum_{i=0}^m\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)+$$

$$\sum_{i=0}^m\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)j(n-m)\kappa(p)$$

Or:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^m\binom{n}{i}\left(1-F(p)\right)^iF(p)^{n-i}(m-i)\right]\geq$$

$$-m^2\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-m\left(1-F(p)\right)\sum_{i=1}^m\sum_{j=0}^{n-m}\binom{m-1}{i-1}\binom{n-m}{j}\left(1-F(p)\right)^{i-1+j}F(p)^{n-i-j}$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)-$$

$$\left(1-F(p)\right)\sum_{k=1}^m\binom{n-1}{k-1}\left(1-F(p)\right)^{k-1}F(p)^{n-k}(m-k)n^2\left(\kappa(p)-m\left(1-F(p)\right)\right)+$$

$$\left(1-F(p)\right)\sum_{i=1}^m\sum_{j=0}^{m-i}\binom{m-1}{i-1}\binom{n-m}{j}\left(1-F(p)\right)^{i-1+j}F(p)^{n-i-j}(m-i-j)m\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)+$$

$$\left(1-F(p)\right)\sum_{i=0}^m\sum_{j=1}^{m-i}\binom{m}{i}\binom{n-m-1}{j-1}\left(1-F(p)\right)^{i+j-1}F(p)^{n-i-j}(m-i-j)(n-m)^2\kappa(p)$$

Or:

$$-m\left[(n(1-F(p))-\kappa(p))^2+F(p)(n(1-F(p))-\kappa(p))\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}(1-F(p))^i F(p)^{n-i}(m-i)\right]\geq$$

$$-m^2(n(1-F(p))-\kappa(p))^2$$

$$-m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}$$

$$\left((k+j+1)n(\kappa(p)-m(1-F(p)))-(k+1)\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-(1-F(p))\sum_{i=1}^{m}\binom{n-1}{i-1}(1-F(p))^{i-1}F(p)^{n-i}(m-i)n^2(\kappa(p)-m(1-F(p)))$$

$$+(1-F(p))\sum_{i=1}^{m}\binom{n-1}{i-1}(1-F(p))^{i-1}F(p)^{n-i}(m-i)m\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+(1-F(p))\sum_{i=1}^{m}\binom{n-1}{i-1}(1-F(p))^{i-1}F(p)^{n-i}(m-i)(n-m)^2\kappa(p)$$

Or:

$$-m\left[(n(1-F(p))-\kappa(p))^2+F(p)(n(1-F(p))-\kappa(p))\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}(1-F(p))^i F(p)^{n-i}(m-i)\right]\geq$$

$$-m^2(n(1-F(p))-\kappa(p))^2$$

$$-m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}(k+j)n(\kappa(p)-m(1-F(p)))$$

$$-m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}n(\kappa(p)-m(1-F(p)))$$

$$+m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}k\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+m(1-F(p))\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}(1-F(p))^{k+j}F(p)^{n-1-k-j}j(n-m)\kappa(p)$$

$$+m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Dividing each side by m:

$$-\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-(n-1)\left(1-F(p)\right)^2\sum_{k=1}^{n-1}\binom{n-2}{k-1}\left(1-F(p)\right)^{k-1}F(p)^{n-1-k}n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$-\left(1-F(p)\right)n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$+(m-1)\left(1-F(p)\right)^2\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-2}{k-1}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+\left(1-F(p)\right)\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+(n-m)\left(1-F(p)\right)^2\sum_{k=0}^{m-1}\sum_{j=1}^{n-m-1}\binom{m-1}{k}\binom{n-m-1}{j-1}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}(n-m)\kappa(p)$$

$$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Or:

$$-\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-(n^2-n)\left(1-F(p)\right)^2\kappa(p)+m(n^2-n)\left(1-F(p)\right)^3$$

$$-\left(1-F(p)\right)n\kappa(p)+nm\left(1-F(p)\right)^2$$

$$+(m-1)\left(1-F(p)\right)\kappa(p)^2-(m^2-m)\left(1-F(p)\right)^2\kappa(p)$$

$$+\kappa(p)^2 - m\left(1-F(p)\right)\kappa(p)$$

$$+(n-m)^2\left(1-F(p)\right)^2\kappa(p)$$

$$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Or:

$$-\left[mF(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)^2 + mF(p)^2\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$+\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2 + F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i) \geq$$

$$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2F(p)$$

$$+\left(m\left(1-F(p)\right)-\kappa(p)\right)\left(n\left(1-F(p)\right)-\kappa(p)\right)F(p)$$

$$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Or:

$$\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2 + F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i) \geq$$

$$\left(m-\kappa(p)\right)\left(n\left(1-F(p)\right)-\kappa(p)\right)F(p)$$

$$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Note that $m-\kappa(p) = m-\lambda(p,p)-m\left(1-F(p)\right) = \sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)$. Thus:

$$\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i) \geq$$

$$\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

It follows that $W_o'(p) \geq W_s'(p)$ if and only if:

$$\sum_{i=0}^{m} \binom{n}{i} (1-F(p))^i F(p)^{n-i}(m-i) - \sum_{i=1}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} F(p)^{n-i}(m-i) \geq 0$$

Using the identity $\binom{n-1}{i} = \binom{n}{i} - \binom{n-1}{i-1}$, this equation becomes:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i F(p)^{n-i}(m-i)$$

$$+\sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^i F(p)^{n-i}(m-i) - \sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} F(p)^{n-i}(m-i) \geq 0$$

This reduces to:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i F(p)^{n-i}(m-i) - \sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} F(p)^{n-i+1}(m-i) \geq 0$$

Because $\binom{n-1}{-1} = 0$, and substituting j for $i-1$, we get:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i F(p)^{n-i}(m-i) - \sum_{j=0}^{m-1} \binom{n-1}{j} (1-F(p))^j F(p)^{n-j}(m-j-1)$$

$$= \sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i F(p)^{n-i} \geq 0.$$

This last statement is clearly true, and the inequality holds strictly if and only if $p > 0$.

Proof of Theorem 2.6.

If we take the derivative of $W_s(c)$ with respect to the transportation cost, c, we get:

$$W_s'(c) = -mf(p+c)\left(E\left[v|v \geq p+c\right] - c\right)$$

$$+m\left(1-F(p+c)\right)\left(\tfrac{f(p+c)}{1-F(p+c)}\left(E\left[v|v \geq p+c\right] - p - c\right) - 1\right) + \tfrac{d}{dc}\lambda\left\{\hat{v}, p+c\right\} E\left[v|v \geq \hat{v}\right]$$

$$+\lambda\left\{\hat{v}, p+c\right\} \tfrac{f(\hat{v})}{1-F(\hat{v})}\left(E\left[v|v \geq \hat{v}\right] - \hat{v}\right)\tfrac{d\hat{v}}{dc} - (n-m)\left(1-F(\hat{v})\right) + (n-m)f(\hat{v})\tfrac{d\hat{v}}{dc}c.$$

After simplifying:

$$W_s'(c) = -mpf(p+c) - m\left(1-F(p+c)\right) - (n-m)\left(1-F(\hat{v})\right) + (n-m)f(\hat{v})\tfrac{d\hat{v}}{dc}c$$

$$-\lambda(\hat{v}, p+c)\tfrac{\hat{v}f(\hat{v})}{1-F(\hat{v})}\tfrac{d\hat{v}}{dc} + E\left[v|v \geq \hat{v}\right]\left[\tfrac{\lambda\{\hat{v},p+c\}f(\hat{v})}{1-F(\hat{v})}\tfrac{d\hat{v}}{dc} + \tfrac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\right].$$

At $c=0$, if we substitute expression (8), this becomes:

$$W_s'(0) = E\left[v|v\geq p\right]\left[(n-m)f(p) + \tfrac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\big|_{c=0}\right] - npf(p) - n\left(1-F(p)\right).$$

From the proof of Theorem 2.1 and substituting expression (7), we get:

$$W_o'(0) = n\left(f(p) + \tfrac{(1-F(p))\kappa'(p)}{\kappa(p)}\right)E\left[v|v\geq p\right] - npf(p) - n\left(1-F(p)\right).$$

Thus, $W_o'(0) \geq W_s'(0)$ if and only if

$$\frac{n\left(1-F(p)\right)\kappa'(p)}{\kappa(p)} \geq \frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\Big|_{c=0} - mf(p). \tag{10}$$

Note that $\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\} =$
$-\frac{f(p+c)}{(1-F(p+c))F(p+c)}\sum_{i=0}^{m}\binom{m}{i}\left(1-F(p+c)\right)^{i}F(p+c)^{m-i}\sum_{j=0}^{n-m}\binom{n-m}{j}\left(1-F(\hat{v})\right)^{j}F(\hat{v})^{n-m-j}$
$\min\{m-i,j\}i + \frac{mf(p+c)}{F(p+c)}\lambda(\hat{v}, p+c)$
$-\frac{f(\hat{v})}{(1-F(\hat{v}))F(\hat{v})}\frac{d\hat{v}}{dc}\sum_{i=0}^{m}\binom{m}{i}\left(1-F(p+c)\right)^{i}F(p+c)^{m-i}\sum_{j=0}^{n-m}\binom{n-m}{j}\left(1-F(\hat{v})\right)^{j}F(\hat{v})^{n-m-j}$
$\min\{m-i,j\}j + \frac{(n-m)f(\hat{v})}{F(\hat{v})}\frac{d\hat{v}}{dc}\lambda(\hat{v}, p+c).$
Evaluated at $c=0$, this becomes: $\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\big|_{c=0} =$
$-\frac{f(p)}{(1-F(p))F(p)}\sum_{i=0}^{m}\binom{m}{i}\sum_{j=0}^{n-m}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}i + \frac{mf(p)}{F(p)}\lambda(p,p)$
$-\frac{f(p)}{(1-F(p))F(p)}\frac{d\hat{v}}{dc}\big|_{c=0}\sum_{i=0}^{m}\binom{m}{i}\sum_{j=0}^{n-m}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}j + \frac{(n-m)f(p)}{F(p)}\frac{d\hat{v}}{dc}\big|_{c=0}\lambda(p,p).$
Using expression (4), and collecting terms, this simplifes to:
$\frac{d}{dc}\lambda\left(\hat{v}, p+c\right)\big|_{c=0} = \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)}\frac{d\hat{v}}{dc}\big|_{c=0}\right)\left(\kappa(p) - m\left(1-F(p)\right)\right)$
$-\frac{f(p)}{(1-F(p))F(p)}\sum_{i=0}^{m}\binom{m}{i}\sum_{j=0}^{n-m}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}\left(i + j\frac{d\hat{v}}{dc}\big|_{c=0}\right).$
Thus expression (10) becomes:
$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} + mf(p) - \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)}\frac{d\hat{v}}{dc}\big|_{c=0}\right)\left(\kappa(p) - m\left(1-F(p)\right)\right) \geq$
$-\frac{f(p)}{(1-F(p))F(p)}\sum_{i=0}^{m}\binom{m}{i}\sum_{j=0}^{n-m}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}\left(i + j\frac{d\hat{v}}{dc}\big|_{c=0}\right).$
Or:
$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} + \left(\frac{mf(p)}{F(p)} + \frac{(n-m)f(p)}{F(p)}\frac{d\hat{v}}{dc}\big|_{c=0}\right)\left(m - \kappa(p)\right) - mf(p)\left(m + (n-m)\frac{d\hat{v}}{dc}\big|_{c=0} - 1\right) \geq$
$-\frac{f(p)}{(1-F(p))F(p)}\sum_{i=0}^{m}\binom{m}{i}\sum_{j=0}^{n-m}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}\left(i + j\frac{d\hat{v}}{dc}\big|_{c=0}\right).$
Evaluated at $c=0$, and using expression (8), this becomes:

$$\frac{d}{dc}\lambda\left\{\hat{v}, p+c\right\}\big|_{c=0} = f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right) + \left(\frac{n-m-j}{F(p)} - \frac{j}{1-F(p)}\right)\frac{(n-m)(1-F(p))}{\lambda(p,p)}\right)\min\{m-i,j\}$$

This simplifies to:

$$\frac{d}{dc}\lambda\left\{\hat{v},p+c\right\}\Big|_{c=0} = f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)}-\frac{i}{1-F(p)}\right)+\left(\frac{n-m-j}{F(p)}-\frac{j}{1-F(p)}\right)\frac{(n-m)(1-F(p))}{\lambda(p,p)}\right)\min\{m-i,j\}$$

Thus $W_o'(0) \geq W_s'(0)$ if and only if:

$$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} \geq f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}$$

$$\left(\left(\frac{m-i}{F(p)}-\frac{i}{1-F(p)}\right)+\left(\frac{n-m-j}{F(p)}-\frac{j}{1-F(p)}\right)\frac{(n-m)(1-F(p))}{\lambda(p,p)}\right)\min\{m-i,j\}-mf(p)$$

Multiplying each side by $\lambda(p,p)\kappa(p)$:

$$\lambda(p,p)n(1-F(p))\kappa'(p) \geq f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(\left(\frac{m-i}{F(p)}-\frac{i}{1-F(p)}\right)\lambda(p,p)\kappa(p)+\left(\frac{n-m-j}{F(p)}-\frac{j+\frac{m}{n-m}}{1-F(p)}\right)\kappa(p)(n-m)(1-F(p))\right)$$

Combining statements (6) and (3):

$$\kappa'(p) = \sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\left(\frac{n-i-j}{F(p)}-\frac{i+j}{1-F(p)}\right)f(p)\min\{m,i+j\}.$$

Rearranging terms and applying statement (3):

$$\kappa'(p) = f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\left(\frac{n-i-j}{F(p)}-\frac{i+j}{1-F(p)}\right)\min\{m-i,j\}+$$

$$mf(p)(1-F(p))\sum_{i=0}^{n-1}\binom{n-1}{i}(1-F(p))^{i}F(p)^{n-1-i}\left(\frac{n-1-i}{F(p)}-\frac{i+1}{1-F(p)}\right).$$

Note that $\sum_{i=0}^{n-1}\binom{n-1}{i}(1-F(p))^{i}F(p)^{n-1-i}\left(\frac{n-1-i}{F(p)}-\frac{i+1}{1-F(p)}\right)=\frac{-1}{1-F(p)}$. Thus:

$$\kappa'(p) = f(p)\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\left(\frac{n-i-j}{F(p)}-\frac{i+j}{1-F(p)}\right)\min\{m-i,j\}-mf(p).$$

Substituting for $\kappa'(p)$ and dividing each side by $f(p)$, it follows that $W_o'(p) \geq W_s'(p)$ if and only if:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}(1-F(p))^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(\left(\frac{n-i-j}{F(p)} - \frac{i+j}{1-F(p)}\right)\lambda(p,p) - m\right) n\left(1-F(p)\right) \geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(\left(\frac{m-i}{F(p)} - \frac{i}{1-F(p)}\right)\lambda(p,p)\kappa(p) + \left(\frac{n-m-j}{F(p)} - \frac{j+\frac{m}{n-m}}{1-F(p)}\right)\kappa(p)(n-m)\left(1-F(p)\right)\right)$$

Multiplying each side by $F(p)$:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\left\{m-i,j\right\}$$

$$\left(n^2\left(1-F(p)\right)\lambda(p,p) - n(i+j)\lambda(p,p) - mn\left(1-F(p)\right)F(p)\right) \geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(m\lambda(p,p) - \frac{i\lambda(p,p)}{1-F(p)} + (n-m)^2\left(1-F(p)\right) - (n-m)j - mF(p)\right)\kappa(p)$$

Rearranging terms:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\left\{m-i,j\right\}$$

$$\left(n^2\left(1-F(p)\right)\lambda(p,p) - mn\left(1-F(p)\right)F(p) + \left(mF(p) - m\lambda(p,p) - (n-m)^2\left(1-F(p)\right)\right)\kappa(p)\right) \geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n(i+j)\lambda(p,p) - \frac{i\lambda(p,p)\kappa(p)}{1-F(p)} - (n-m)j\kappa(p)\right)$$

Using the substitution in statement (4) and cancelling terms:

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\left\{m-i,j\right\}$$

$$\left[-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2 - mF(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right] \geq$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\{m-i,j\}$$

$$\left(n(i+j)\left(\kappa(p) - m\left(1-F(p)\right)\right) - \frac{i(\kappa(p)-m(1-F(p)))\kappa(p)}{1-F(p)} - (n-m)j\kappa(p)\right)$$

Note that $\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}\min\left\{m-i,j\right\}X_{ij}$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i)X_{ij}$$

$$-\sum_{i=0}^{m}\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)X_{ij}.$$

It follows that $W_o'(p) \geq W_s'(p)$ if and only if:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right] \geq$$

$$m\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-\sum_{i=0}^{m}\sum_{j=0}^{n-m}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}i$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-\sum_{i=0}^{m}\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)(i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)+$$

$$\sum_{i=0}^{m}\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)+$$

$$\sum_{i=0}^{m}\sum_{j=0}^{m-i}\binom{m}{i}\binom{n-m}{j}\left(1-F(p)\right)^{i+j}F(p)^{n-i-j}(m-i-j)j(n-m)\kappa(p)$$

Or:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right] \geq$$

$$-m^2\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-m\left(1-F(p)\right)\sum_{i=1}^{m}\sum_{j=0}^{n-m}\binom{m-1}{i-1}\binom{n-m}{j}\left(1-F(p)\right)^{i-1+j}F(p)^{n-i-j}$$

$$\left((i+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-i\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)-$$

$$\left(1-F(p)\right)\sum_{k=1}^{m}\binom{n-1}{k-1}\left(1-F(p)\right)^{k-1}F(p)^{n-k}(m-k)n^2\left(\kappa(p)-m\left(1-F(p)\right)\right)+$$

$$\left(1-F(p)\right)\sum_{i=1}^{m}\sum_{j=0}^{m-i}\binom{m-1}{i-1}\binom{n-m}{j}\left(1-F(p)\right)^{i-1+j}F(p)^{n-i-j}(m-i-j)m\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)+$$

$$\left(1-F(p)\right)\sum_{i=0}^{m}\sum_{j=1}^{m-i}\binom{m}{i}\binom{n-m-1}{j-1}\left(1-F(p)\right)^{i+j-1}F(p)^{n-i-j}(m-i-j)(n-m)^2\kappa(p)$$

Or:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$$-m^2\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}$$

$$\left((k+j+1)n\left(\kappa(p)-m\left(1-F(p)\right)\right)-(k+1)\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)-j(n-m)\kappa(p)\right)$$

$$-\left(1-F(p)\right)\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)n^2\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$+\left(1-F(p)\right)\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)m\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+\left(1-F(p)\right)\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)(n-m)^2\kappa(p)$$

Or:

$$-m\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$$-m^2\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}(k+j)n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$-m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$+m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}k\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+m\left(1-F(p)\right)\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-1}{k}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}j(n-m)\kappa(p)$$

$$+m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Dividing each side by m:

$$-\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$$

$$-(n-1)\left(1-F(p)\right)^2\sum_{k=1}^{n-1}\binom{n-2}{k-1}\left(1-F(p)\right)^{k-1}F(p)^{n-1-k}n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$-\left(1-F(p)\right)n\left(\kappa(p)-m\left(1-F(p)\right)\right)$$

$$+(m-1)\left(1-F(p)\right)^2\sum_{k=0}^{m-1}\sum_{j=0}^{n-m}\binom{m-2}{k-1}\binom{n-m}{j}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+\left(1-F(p)\right)\left(\frac{\kappa(p)^2}{1-F(p)}-m\kappa(p)\right)$$

$$+(n-m)\left(1-F(p)\right)^2\sum_{k=0}^{m-1}\sum_{j=1}^{n-m-1}\binom{m-1}{k}\binom{n-m-1}{j-1}\left(1-F(p)\right)^{k+j}F(p)^{n-1-k-j}(n-m)\kappa(p)$$

$$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^{m}\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$$

Or:

$$-\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$$

$$\left[mF(p)-\sum_{i=0}^{m}\binom{n}{i}\left(1-F(p)\right)^{i}F(p)^{n-i}(m-i)\right]\geq$$

$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2$

$-(n^2-n)\left(1-F(p)\right)^2\kappa(p)+m(n^2-n)\left(1-F(p)\right)^3$

$-\left(1-F(p)\right)n\kappa(p)+nm\left(1-F(p)\right)^2$

$+(m-1)\left(1-F(p)\right)\kappa(p)^2-(m^2-m)\left(1-F(p)\right)^2\kappa(p)$

$+\kappa(p)^2-m\left(1-F(p)\right)\kappa(p)$

$+(n-m)^2\left(1-F(p)\right)^2\kappa(p)$

$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^m\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$

Or:

$-\left[mF(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+mF(p)^2\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$

$+\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$

$\sum_{i=0}^m\binom{n}{i}\left(1-F(p)\right)^iF(p)^{n-i}(m-i)\geq$

$-m\left(n\left(1-F(p)\right)-\kappa(p)\right)^2F(p)$

$+\left(m\left(1-F(p)\right)-\kappa(p)\right)\left(n\left(1-F(p)\right)-\kappa(p)\right)F(p)$

$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^m\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$

Or:

$\left[\left(n\left(1-F(p)\right)-\kappa(p)\right)^2+F(p)\left(n\left(1-F(p)\right)-\kappa(p)\right)\right]$

$\sum_{i=0}^m\binom{n}{i}\left(1-F(p)\right)^iF(p)^{n-i}(m-i)\geq$

$(m-\kappa(p))\left(n\left(1-F(p)\right)-\kappa(p)\right)F(p)$

$+\left(n\left(1-F(p)\right)-\kappa(p)\right)^2\sum_{i=1}^m\binom{n-1}{i-1}\left(1-F(p)\right)^{i-1}F(p)^{n-i}(m-i)$

Note that $m - \kappa(p) = m - \lambda(p,p) - m\,(1-F(p)) = \sum_{i=0}^{m} \binom{n}{i} (1-F(p))^i \, F(p)^{n-i} (m-i)$. Thus:

$$(n\,(1-F(p)) - \kappa(p))^2 \sum_{i=0}^{m} \binom{n}{i} (1-F(p))^i \, F(p)^{n-i}(m-i) \geq$$

$$(n\,(1-F(p)) - \kappa(p))^2 + (n\,(1-F(p)) - \kappa(p))^2 \sum_{i=1}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} \, F(p)^{n-i}(m-i)$$

It follows that $W_o'(p) \geq W_s'(p)$ if and only if:

$$\sum_{i=0}^{m} \binom{n}{i} (1-F(p))^i \, F(p)^{n-i}(m-i) - \sum_{i=1}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} \, F(p)^{n-i}(m-i) \geq 0$$

Using the identity $\binom{n-1}{i} = \binom{n}{i} - \binom{n-1}{i-1}$, this equation becomes:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i \, F(p)^{n-i}(m-i)$$

$$+ \sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^i \, F(p)^{n-i}(m-i) - \sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} \, F(p)^{n-i}(m-i) \geq 0$$

This reduces to:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i \, F(p)^{n-i}(m-i) - \sum_{i=0}^{m} \binom{n-1}{i-1} (1-F(p))^{i-1} \, F(p)^{n-i+1}(m-i) \geq 0$$

Because $\binom{n-1}{-1} = 0$, and substituting j for $i-1$, we get:

$$\sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i \, F(p)^{n-i}(m-i) - \sum_{j=0}^{m-1} \binom{n-1}{j} (1-F(p))^j \, F(p)^{n-j}(m-j-1)$$

$$= \sum_{i=0}^{m} \binom{n-1}{i} (1-F(p))^i \, F(p)^{n-i} \geq 0.$$

This last statement is clearly true, and the inequality holds strictly if and only if $p > 0$.